Factsheet: Gamma distribution

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Summary

A factsheet for the gamma distribution.

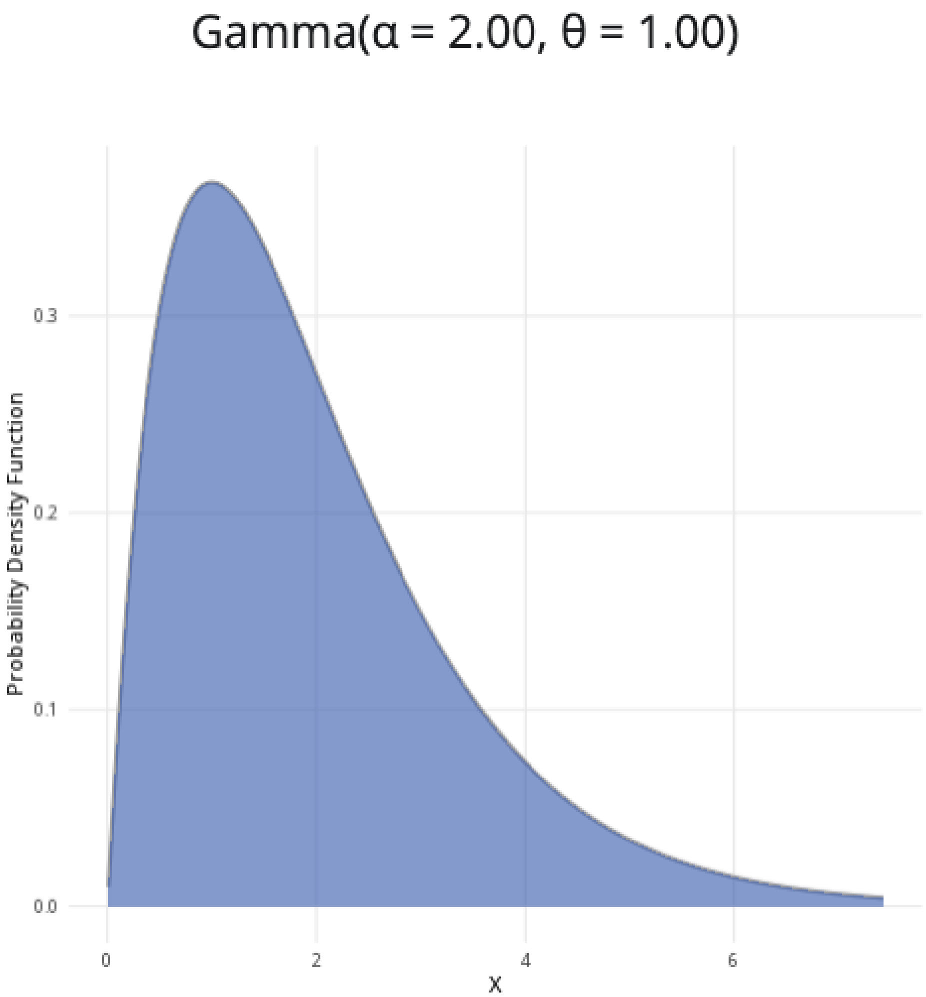

Figure 1: An example of the gamma distribution with $\alpha = 2$ and $\theta = 1$.

Where to use: The gamma distribution generalizes the exponential distribution, allowing for greater or lesser variance. It is used to model positive continuous random variables that have skewed distributions.

Notation: $X \sim \mathrm{Gamma}(\alpha, \theta)$ or $X \sim \mathrm{Gam}(\alpha, \theta)$

Parameters: Two real numbers α and θ, which are related to the mean μ and variance σ^2:

- $\alpha = \frac{\mu^2}{\sigma^2}$ (shape parameter)
- $\theta = \frac{\sigma^2}{\mu}$ (scale parameter)

Quantity	Value	Notes
Mean	$\mathbb{E}(X) = \alpha\theta$	
Variance	$\mathbb{V}(X) = \alpha\theta^2$	
PDF	$\mathbb{P}(X = x) = \frac{x^{\alpha-1}\exp\left(-\frac{x}{\theta}\right)}{\Gamma(\alpha)\theta^\alpha}$	$\Gamma(x)$ the gamma function of x
CDF	$\mathbb{P}(X \leq x) = \frac{\mathrm{Gam}\left(\alpha, \frac{x}{\theta}\right)}{\Gamma(\alpha)}$	$\mathrm{Gam}(\alpha, \theta)$ is the PDF of the gamma distribution

Example: You collect historical data on the time to failure of a machine from Cantor's Confectionery. The mean is 83 days and the variance is 50.3. You can then use this to estimate the shape and scale parameters of the gamma distribution:

- $\alpha = \frac{83^2}{50.3} = 136.958250497 \approx 137$
- $\theta = \frac{50.3}{83} = 0.60602409638 \approx 0.61$

The distribution can be expressed as $X \sim \mathrm{Gam}(137, 0.61)$, where the shape parameter is 137 and the scale parameter is 0.61.

Further reading

This interactive element appears in Overview: Probability distributions. Please click this link to go to the guide.

Version history

v1.0: initial version created 04/25 by tdhc and Michelle Arnetta as part of a University of St Andrews VIP project.

- v1.1: moved to factsheet form and populated with material from Overview: Probability distributions by tdhc.