Factsheet: Discrete uniform distribution

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Summary

A factsheet for the discrete uniform distribution.

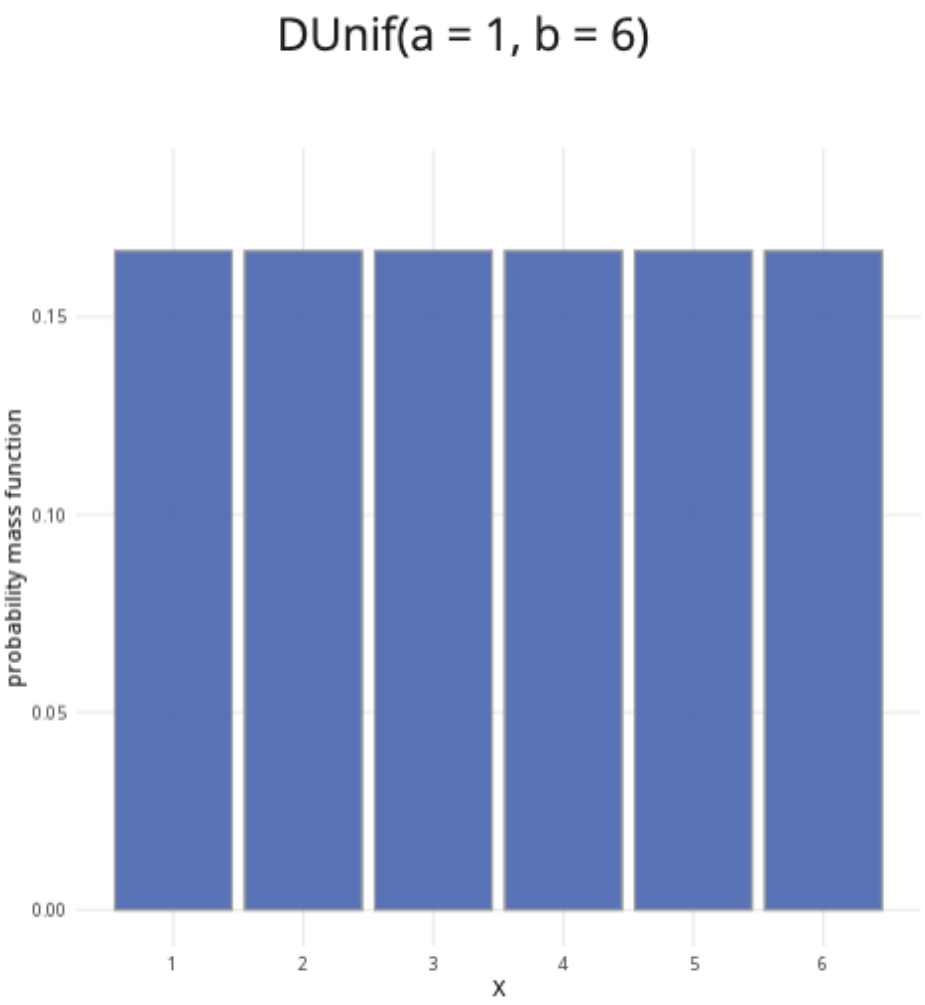

Figure 1: An example of the discrete uniform distribution with $a = 1$ and $b = 6$.

Where to use: The discrete uniform distribution is used when all integer outcomes x in the interval a to b are equally likely. X is a random variable for integer outcomes x where for $a \leq x \leq b$, and the probability of each outcome $1/n$, where $n = b - a + 1$.

Notation: $X \sim \text{Uniform}(a, b)$ or $X \sim U(a, b)$

Parameters: The numbers a, b are integers where

- a is the minimum value of an outcome
- b is the maximum value of an outcome

There are n outcomes in total, with $n = b - a + 1$.

Quantity	Value	Notes
Mean	$\mathbb{E}(X) = \frac{a+b}{2}$.	
Variance	$\mathbb{V}(X) = \frac{n^2-1}{12}$.	
PMF	$\mathbb{P}(X = x) = \frac{1}{n}$	
CDF	$\mathbb{P}(X \leq x) = \begin{cases} 0 & \text{if } x \leq a \\ \frac{\lfloor x \rfloor - a + 1}{n} & \text{if } a < x < b \\ 1 & \text{if } x \geq b \end{cases}$	$\lfloor x \rfloor$ is the **floor function**

Example: You roll a fair six-sided die, where all outcomes $(1, 2, 3, 4, 5,$ and $6)$ are equally likely. This can be expressed as $X \sim U(1, 6)$. It means 1 is the minimum value and 6 is the maximum value, where all discrete values of X for $1 \leq x \leq 6$ are equally likely.

Further reading

This interactive element appears in Overview: Probability distributions.

Version history

v1.0: initial version created 08/25 by tdhc.