

Factsheet: Chi-squared distribution

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Summary

A factsheet for the χ^2 distribution.

Chi-Squared(k = 5)

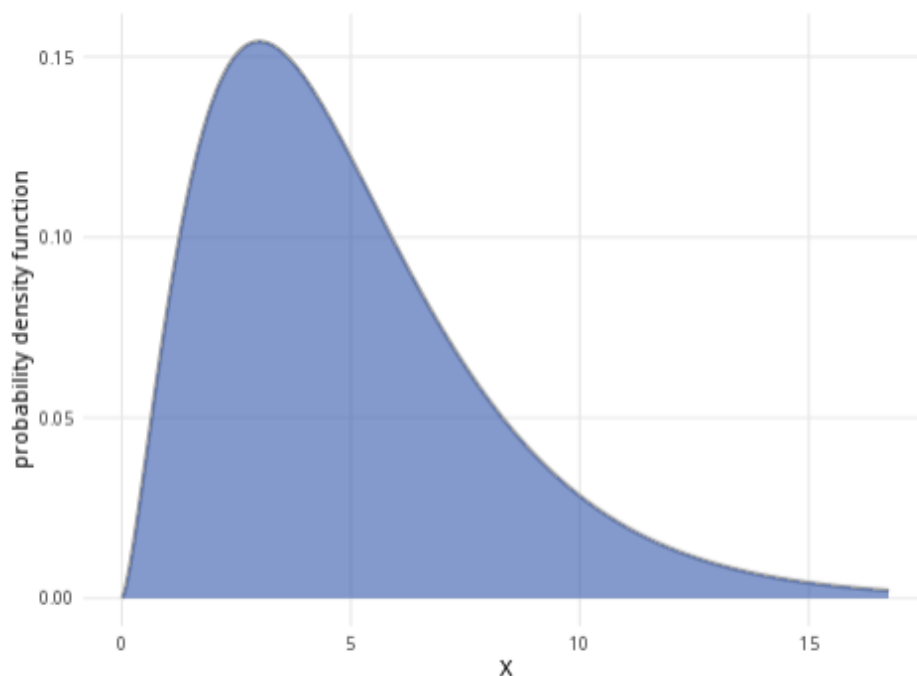


Figure 1: An example of the chi-squared distribution with $k = 5$.

Where to use: The χ^2 distribution is used for hypothesis testing, such as for goodness of fit tests and tests for independence. (See [Guide: Introduction to hypothesis testing](#) for more.) It is a special case of the gamma distribution, as $\chi^2(k) = \text{Gam}(\frac{k}{2}, 2)$.

Notation: $X \sim \chi^2(k)$

Parameter: The integer k is the number of degrees of freedom in the sample.

Quantity	Value	Notes
Mean	$\mathbb{E}(X) = k$	

Quantity	Value	Notes
Variance	$V(X) = 2k$	
PDF	$\mathbb{P}(X = x) = \frac{x^{\frac{k}{2}-1} \exp(-\frac{x}{2})}{2^{\frac{k}{2}} \Gamma(\frac{k}{2})}$	$\Gamma(x)$ is the gamma function
CDF	$\mathbb{P}(X \leq x) = \frac{1}{\Gamma(\frac{k}{2})} \text{Gam}(\frac{k}{2}, \frac{x}{2})$	$\Gamma(x)$ is the gamma function, $\text{Gam}(\alpha, \theta)$ is the PDF of the gamma distribution

Examples:

- **Goodness of fit example:** You have a six-sided die with six possible outcomes: 1, 2, 3, 4, 5, and 6. You calculate the expected frequencies of each outcome. Then you roll the die many times and record the observed frequencies of each outcome. Since there are 6 categories,

$$\text{degrees of freedom} = \text{number of categories} - 1 = 6 - 1 = 5$$

This can be expressed as $X \sim \chi^2(5)$, meaning the degrees of freedom is 5.

- **Test for independence example:** You are investigating whether there is a correlation between two variables: candy colour and flavour. You have 5 categories of colours and 3 categories of flavours. Calculating the degrees of freedom can be done with the formula:

$$(\text{categories of colours} - 1)(\text{categories of flavours} - 1) = (5 - 1)(3 - 1) = (4)(2) = 8.$$

You can model $X \sim \chi^2(8)$, meaning that there are 8 degrees of freedom.

Further reading

This interactive element appears in [Overview: Probability distributions](#). Please [click this link to go to the guide](#).

Version history

v1.0: initial version created 04/25 by tdhc and Michelle Arnetta as part of a University of St Andrews VIP project.

- v1.1: moved to factsheet form and populated with material from [Overview: Probability distributions](#) by tdhc.

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