

Answers: Introduction to partial differentiation

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Summary

Answers to questions relating to the guide on the introduction to partial differentiation.

These are the answers to [Questions: Introduction to partial differentiation](#).

Please attempt the questions before reading these answers!

Q1

$$1.1. \quad \frac{\partial f}{\partial x} = 2xy \text{ and } \frac{\partial f}{\partial y} = x^2 + 3y^2.$$

$$1.2. \quad \frac{\partial f}{\partial x} = 9x^2 + y \text{ and } \frac{\partial f}{\partial y} = x - 8y^3.$$

$$1.3. \quad \frac{\partial f}{\partial x} = 2y \cos(2x) \text{ and } \frac{\partial f}{\partial y} = \sin(2x).$$

$$1.4. \quad \frac{\partial f}{\partial x} = ye^{xy} + 4xy^3 \text{ and } \frac{\partial f}{\partial y} = xe^{xy} + 6x^2y^2.$$

$$1.5. \quad \frac{\partial f}{\partial x} = \frac{1}{x} + \ln(y) + 3 \text{ and } \frac{\partial f}{\partial y} = \frac{x}{y}.$$

$$1.6. \quad \frac{\partial f}{\partial x} = -\frac{y}{x^2} - \frac{1}{y} \text{ and } \frac{\partial f}{\partial y} = \frac{1}{x} + \frac{x}{y^2}.$$

$$1.7. \quad \frac{\partial f}{\partial x} = \exp(y^2) \text{ and } \frac{\partial f}{\partial y} = 2xy \exp(y^2).$$

$$1.8. \quad \frac{\partial f}{\partial x} = \frac{x}{\sqrt{x^2 + y^2}} \text{ and } \frac{\partial f}{\partial y} = \frac{y}{\sqrt{x^2 + y^2}}.$$

$$1.9. \quad \frac{\partial f}{\partial x} = 12(3x + 2y)^3 \text{ and } \frac{\partial f}{\partial y} = 8(3x + 2y)^3.$$

$$1.10. \quad \frac{\partial f}{\partial x} = y^2 \cos(xy) \text{ and } \frac{\partial f}{\partial y} = x \cos(xy) - x^2y \sin(xy).$$

$$1.11. \quad \frac{\partial f}{\partial x} = 2x \cos(x^2 + y^2) \text{ and } \frac{\partial f}{\partial y} = 2y \cos(x^2 + y^2).$$

$$1.12. \quad \frac{\partial f}{\partial x} = \frac{2xy^2}{1 + x^2y^2} \text{ and } \frac{\partial f}{\partial y} = \frac{2x^2y}{1 + x^2y^2}.$$

$$1.13. \quad \frac{\partial f}{\partial x} = 2xy \sin(z) \text{ and } \frac{\partial f}{\partial y} = x^2 \sin(z) \text{ and } \frac{\partial f}{\partial z} = x^2 y \cos(z).$$

$$1.14. \quad \frac{\partial f}{\partial x} = (y+z)(2x+y+z) \text{ and } \frac{\partial f}{\partial y} = (x+z)(x+2y+z) \text{ and } \frac{\partial f}{\partial z} = (x+y)(x+y+2z).$$

$$1.15. \quad \frac{\partial f}{\partial x} = \frac{yz(y+z)}{(x+y+z)^2} \text{ and } \frac{\partial f}{\partial y} = \frac{xz(x+z)}{(x+y+z)^2} \text{ and } \frac{\partial f}{\partial z} = \frac{xy(x+y)}{(x+y+z)^2}.$$

Q2

$$2.1. \quad \frac{\partial^2 f}{\partial x^2} = 2 \text{ and } \frac{\partial^2 f}{\partial y^2} = -2 \text{ so } \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0.$$

$$2.2. \quad \frac{\partial^2 f}{\partial x^2} = 0 \text{ and } \frac{\partial^2 f}{\partial y^2} = 0 \text{ so } \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0.$$

$$2.3. \quad \frac{\partial^2 f}{\partial x^2} = 6x \text{ and } \frac{\partial^2 f}{\partial y^2} = -6x \text{ so } \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0.$$

$$2.4. \quad \frac{\partial^2 f}{\partial x^2} = -\cos(x) \sinh(y) \text{ and } \frac{\partial^2 f}{\partial y^2} = \cos(x) \sinh(y) \text{ so } \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0.$$

$$2.5. \quad \frac{\partial^2 f}{\partial x^2} = e^x \sin(y) \text{ and } \frac{\partial^2 f}{\partial y^2} = -e^x \sin(y) \text{ so } \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0.$$

$$2.6. \quad \frac{\partial^2 f}{\partial x^2} = \frac{2xy}{(x^2 + y^2)^2} \text{ and } \frac{\partial^2 f}{\partial y^2} = -\frac{2xy}{(x^2 + y^2)^2} \text{ so } \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0.$$

$$2.7. \quad \frac{\partial^2 f}{\partial x^2} = \frac{2(y^2 - x^2)}{(x^2 + y^2)^2} \text{ and } \frac{\partial^2 f}{\partial y^2} = \frac{2(x^2 - y^2)}{(x^2 + y^2)^2} \text{ so } \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0.$$

Q3

$$3.1. \quad \frac{\partial^2 f}{\partial x \partial y} = 2x + 2y \text{ and } \frac{\partial^2 f}{\partial y \partial x} = 2x + 2y \text{ so } \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}.$$

$$3.2. \quad \frac{\partial^2 f}{\partial x \partial y} = -4x \sin(y) \text{ and } \frac{\partial^2 f}{\partial y \partial x} = -4x \sin(y) \text{ so } \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}.$$

$$3.3. \quad \frac{\partial^2 f}{\partial x \partial y} = 20(x+y)^3 \text{ and } \frac{\partial^2 f}{\partial y \partial x} = 20(x+y)^3 \text{ so } \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}.$$

$$3.4. \quad \frac{\partial^2 f}{\partial x \partial y} = -\frac{1}{(y+1)^2} \text{ and } \frac{\partial^2 f}{\partial y \partial x} = -\frac{1}{(y+1)^2} \text{ so } \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}.$$

$$3.5. \quad \frac{\partial^2 f}{\partial x \partial y} = -\frac{xy}{(x^2 + y^2)^{3/2}} \text{ and } \frac{\partial^2 f}{\partial y \partial x} = -\frac{xy}{(x^2 + y^2)^{3/2}} \text{ so } \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}.$$

$$3.6. \quad \frac{\partial^2 f}{\partial x \partial y} = 2x \cos(y) - 2y \sin(x) \text{ and } \frac{\partial^2 f}{\partial y \partial x} = 2x \cos(y) - 2y \sin(x) \text{ so } \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}.$$

$$3.7. \quad \frac{\partial^2 f}{\partial x \partial y} = \frac{1 - (xy)^2}{(1 + (xy)^2)^2} \text{ and } \frac{\partial^2 f}{\partial y \partial x} = \frac{1 - (xy)^2}{(1 + (xy)^2)^2} \text{ so } \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}.$$



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v1.0: initial version created 05/25 by Donald Campbell as part of a University of St Andrews VIP project.

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